APPLICATION OF GEOMETRIC CONSTRAINT PROGRAMMING TO THE KINEMATIC DESIGN OF THREE-POINT HITCHES

S. S. Ambike, J. P. Schmiedeler

ABSTRACT. *Design of three-point hitch systems used with agricultural tractors is quite evolved and is governed by an established standard. Freedom within the standard, though, can be exploited to tailor the individual hitch performance. A hitch can be treated as a four-bar linkage in the vertical longitudinal plane, yet it presents a complicated kinematic synthesis problem because the constraint set imposed by the standard is large and complex. This article proposes the use of Geometric Constraint Programming (GCP) as a design tool to address this problem. GCP uses the drafting mode of commercially available parametric CAD software to impose geometric constraints on objects to define kinematic diagrams. The software then allows the manipulation of the design parameters of the mechanism while the diagram is dynamically updated to satisfy all imposed constraints. GCP is particularly effective in three-point hitch design since a graphical representation of the complex constraint set is obtained, enabling real-time visualization of the interactions between constraints and the effects of varying various design parameters on the design solution. An example is presented to demonstrate the technique, and geometric insight in the form of an implied constraint is uncovered for the chosen model, highlighting the strength of the approach.*

Keywords. *Three-point hitch, Agricultural tractor hitch, Kinematic design, Geometric constraint programming.*

Existing work on the three-point hitch system used with agricultural tractors includes development of hitch dynamometers for the measurement of tractor performance parameters such as draft and drawbar power (Garner et al., 1988). Engineers have sought to better control the implement by controlling three-point hitch motion during farming operations (Cordesses et al., 2002; Lang and Harms, 2002). With the advent of precision agriculture using global positioning guidance systems and steering control, tighter control of the implement path is desired.

The design of the three-point hitch is quite evolved. Morling (1979) gives a comprehensive kinematic and dynamic analysis of the system, and the governing standard is well established (*ASAE Standards*, 2001). The standard, as it is referred to throughout this article, provides freedom that the designer can utilize to fine tune the kinematic design of the hitch system for better control of the implement. This article presents a design tool that can be used to generate configurations suited to specific tractor designs and tailored for particular performance requirements. The set of kinematic constraints imposed by the standard is rather large and complex, with constraints interacting in both obvious and non-obvious ways. This article proposes the use of geometric constraint programming (GCP), first introduced by Kinzel et al. (2006), to effectively use the design freedom to generate kinematic configurations that satisfy the constraints imposed by the standard and the tractor design and to achieve the desired performance. With the kinematic layout established using this technique, the designer can proceed to address the dynamic issues of the hitch design. The designer is equipped with knowledge of the degree of layout manipulation still possible within the standard's limits and a means of quickly evaluating whether dynamics-driven changes to the kinematic layout result in violation of the standard's specifications.

In the vertical longitudinal plane, the three-point hitch system is a six-bar mechanism that can be modeled as two distinct four-bar linkages sharing two links. The first four-bar is called the driving mechanism and powers operation of the hitch system. Referring to figure 1, the driving four-bar linkage consists of the crank (CD), lift rod (DE), lower link (A1E), and tractor body. The other four-bar, called the three-point hitch, also has the tractor as the fixed link (A1B1). The implement mast is the coupler, and the upper link (B1B2) and two lower links taken together (A1A2) form the other two links. In traditional kinematic synthesis problems, constraints have the form of precision positions of links, and the dimensions of all the links are determined to ensure that the relevant link passes through the prescribed positions (Waldron and Kinzel, 2004). In the present problem, the constraints are of a different nature and do not specify precise positions of the links. GCP is a powerful technique that is particularly useful for dealing with kinematic problems involving such "non-standard" constraints. With GCP, the constraint set and performance parameters of the hitch are represented graphically. Because the constraints dynamically drive the design, the designer can see the effects of varying a particular parameter on all other related parameters in real-time. GCP offers greater flexibility in the design process, better development of intuition for the designer, and more rapid solution generation without compromising accuracy as compared to traditional methods of kinematic synthesis.

Submitted for review in November 2005 as manuscript number PM 6175; approved for publication by the Power & Machinery Division of ASABE in September 2006.

The authors are **Satyajit S. Ambike,** Graduate Student, and **James P. Schmiedeler,** Assistant Professor, Department of Mechanical Engineering, Ohio State University, Columbus Ohio. **Corresponding author:** Satyajit S. Ambike, Scott Laboratory, 201 W. 19th Avenue, Columbus, OH 43210; phone: 614-292-4577; fax: 614-292-3163; e-mail: ambike.1@osu.edu.

ISSN 0883−8542

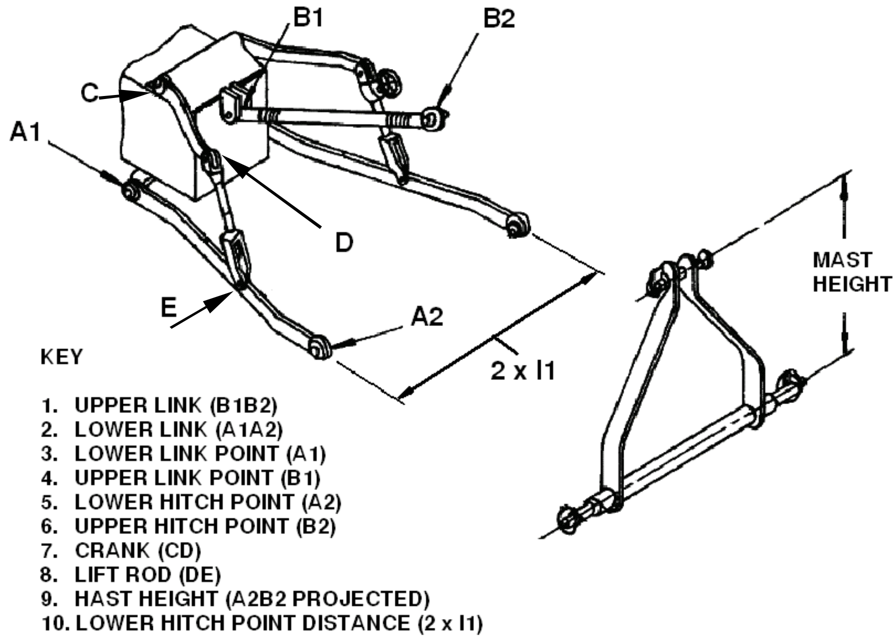

Figure 1. Components of the three-point hitch. (Adapted from *ASAE Standards,* 2001)

METHODS

This section first introduces the GCP technique and then outlines a procedure to create the kinematic diagram of a hitch.

GEOMETRIC CONSTRAINT PROGRAMMING

Geometric constraint programming uses the drafting mode of commercially available parametric computer-aided design (CAD) software to create kinematic diagrams. Most modern parametric CAD software packages allow the user to impose geometric constraints on objects in the sketching mode. The constraints typically appear as graphical icons, each denoted with a symbol that indicates its type. The user selects the constraint icon and then selects relevant entities, such as lines or curves, to apply the constraint. The relationship imposed by the constraint, then, is maintained until the user deletes it. Thus, with GCP, the designer "programs" the mechanism synthesis procedure through the application of geometrical constraints that define the problem with the CAD package's graphical user interface (GUI). In this article, the kinematic layout of a three-point hitch system is created, and the set of constraints specified within the standard is imposed on the elements that define the hitch. SolidEdge (2005) has been used to generate figures 2 through 5 in this article, but the described techniques are equally applicable in most other CAD packages. The specific names used to identify each constraint may vary across different software, but a generic naming convention is as follows:

- Strong Connect: A point is constrained to coincide with another point. In this article, it is assumed that this constraint is automatically applied when a line is drawn from the endpoint or midpoint of another line. A pin joint is defined using Strong Connect.
- Weak Connect: A single point is constrained to lie on a line or a curve. In this article, it is assumed that this constraint is automatically applied when a line is drawn from any point on a line or curve other than the endpoint or midpoint.
- Dimension Lock: A linear or angular dimension is constrained to have a fixed value. This constraint differs from the general dimensioning of unconstrained lengths and angles that simply measure these variable quantities.
- Position Lock: A point, line, or curve is constrained to be fixed relative to the ground (i.e. a fixed reference frame). In this article, imposing a Position Lock on a line (or arc) is assumed to also Dimension Lock its length (or radius).
- Equality: Two lines (or arcs) are constrained to be of equal length (or radius).
- Parallelism: A line is constrained to be parallel to another line. This constraint, along with Strong Connect can be used to form a collinearity constraint. In some CAD packages, horizontal and vertical constraints are special cases that constrain a line to be parallel to one axis of the reference frame.
- Perpendicularity: A line is constrained to be perpendicular to another line.
- Tangency: A curve or a line is constrained to be tangent to another curve or line.

If some parameter (e.g. length, position) of one of the objects that compose a mechanism is varied, the software dynamically updates the entire set of objects so that all of the imposed constraints are satisfied. If the designer attempts to change a parameter in such a fashion that not all of the constraints can be satisfied, an error message is generally created, and the change is not applied. This capability can be exploited to study the effects of varying different parameters in the design. GCP thus provides an intuitive method for designing mechanisms. Since GCP utilizes existing capabilities of commercial CAD software already commonly available to a designer, its implementation requires no additional software cost or development time. The method provides the accuracy and repeatability of analytical synthesis techniques since it leverages the numerical solvers integrated within the software to impose the constraints, but it also provides the geometric insight associated with graphical synthesis techniques since the constraints are specified through the software's GUI.

Three-Point Hitch Design Procedure Using GCP

The kinematic design of the three-point hitch involves establishing the geometric performance parameters listed in table 1 and specifying the spatial location of the link and hitch points that satisfy the standard's constraints. The performance parameters such as minimum transport height and transport pitch have limiting values or permissible zones. Hence, these parameters can be varied during the design process, as can the locations of the hitch points on the tractor. Varying one element affects several others since relationships exist between them. Application of GCP to three-point hitch design involves constructing a kinematic diagram that contains the framework of the hitch and driving linkage and geometrical constructions indicating the positions of the performance parameters and representing the imposed limits. The standard imposes constraints on the three-point hitch only. The driving four-bar is configured to facilitate the desired motion of the hitch. The standard influences the design of the driving four-bar linkage only indirectly.

Three-point hitch systems are divided into categories based on the power take-off (PTO) capacity of the tractor and hence, the size of the hitch system. The constraint set imposed by the standard is isomorphic such that all categories have the same imposed constraints, but the limiting values differ for each category. A constraint set is imposed on a kinematic diagram by dimensioning geometrical constructions to have the limiting values corresponding to a particular category. This diagram can be modified for a different category by simply altering the magnitudes of the dimensions. Separate files may be used as templates for the design of hitches of each category. The following description outlines a procedure to create one such kinematic diagram that describes the three-point hitch completely by defining the positions of the link and hitch points, the lengths of the upper and lower links and the corresponding performance parameters. Furthermore, it determines the lift of the driving linkage crank. The diagram can be enhanced to obtain the complete description of the driving linkage also, thus describing the entire system. The diagram generated assumes that the rear tire is undeformed under load, but the procedure can be carried out for the maximum rated load of the tractor, taking into account the shift of the axle center toward the ground.

Due to the large number of constraints, working with representations of all of them displayed simultaneously can be cumbersome and confusing. Separating various elements of the diagram by drawing them on different layers assists in hitch design by giving the designer freedom to control the amount of visual information displayed on the screen at any one time. The layers are named to indicate their content and arranged sequentially to provide a logical flow. The actual hitch design need not follow this structural flow, but rather can be iterative. It is desirable to put all of the dimensions associated with the entities of a particular layer in a separate, corresponding (dimensions) layer so that they can be hidden to further reduce the clutter. The exceptions are the dimensions actively required in the design process (e.g. transport pitch angle). The procedure outlined here assumes use of such dimension layers, and no further mention of these layers is included. The required dimensions for the construction of the kinematic diagram that are not listed in table 1 are given in table 2.

Layer 1: Basic Structure and Primary Performance Parameters

Layer 1 includes the datum elements and the limiting positions of the primary performance parameters for the hitch to be designed. The specific limiting values of the performance parameters are obtained from the standard for the appropriate hitch category, as indicated in table 1. All elements of Layer 1 are shown in figure 2.

A horizontal line representing the ground and a circle representing the tractor's rear tire, in the side view, form the datum elements fixed to the drawing plane by Position Locks. The circle is further constrained to be Tangent to the ground, and its diameter is Dimension Locked (table 2, No. 1). Four horizontal lines are drawn to indicate the limiting values of the transport height, lower hitch point height, and highest and lowest mast adjustment heights, as defined in the standard. These lines are Dimension Locked to be at the specified heights above the ground (table 1, Nos. 1, 2, and 3). A circle concentric to the tire is drawn with its diameter (table 1, No. 4) Dimension Locked to be the lower hitch point clearance limit. The axle center is indicated by drawing horizontal and vertical center lines from the center of the tire and Position Locking them. To indicate the location of the end of the PTO shaft, a circle of small, arbitrary radius is drawn and Dimension Locked (table 2, No. 2) with respect to the axle center. The center of the circle indicates the position of the end of the PTO shaft of the tractor.

Layer 2: Setting Primary Performance Parameters

Layer 2 allows the designer to choose values for four parameters: transport height, highest and lowest mast

Table 1. Performance parameters for the three-point hitch.

No	Performance Parameter	Drawn in	Source for Limiting Value or Recommended Range
1	Transport height	Layer 1. Basic structure and primary performance parameters	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)
2	Lower hitch point height	Layer 1. Basic structure and primary performance parameters	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)
3	Mast adjustment heights	Layer 1. Basic structure and primary performance parameters	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)
4	Lower hitch point clearance	Layer 1. Basic structure and primary performance parameters	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)
5	Transport pitch	Layer 3. Mechanism side view	*ASAE Standards* S217.12 DEC01 (p. 86)
6	Vertical convergence distance	Layer 3. Mechanism side view	*ASAE Standards* S217.12 DEC01 (p. 88)
7	Horizontal convergence distance	Layer 4. Mechanism top view	*ASAE Standards* S217.12 DEC01 (p. 88)
8	Movement range	Layer 6. Movement Range	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)
9	Leveling adjustment	Layer 6. Movement Range	*ASAE Standards* S217.12 DEC01 (p. 86 table 3)

Table 2. Dimensions required for the construction of the kinematic diagram.

No.	Dimension	Drawn / Used in Layer	Source
1	Diameter of rear tire of the tractor	Layer 1. Basic structure and primary performance parameters	Various
2	Location dimensions of the PTO shaft from the axle center	Layer 1. Basic structure and primary performance parameters	Various
3	Permissible range for the length of the lower link in the side view	Layer 3. Mechanism side view	*ASAE Standards,* S217.12 DEC01 (p. 86, table 2)
4	Mast height	Layer 3. Mechanism side view	*ASAE Standards,* S217.12 DEC01 (p. 88, table 4)
5	Location dimensions of the lower hitch points in the top view	Layer 4. Mechanism top view	*ASAE Standards,* S217.12 DEC01 (p. 86, table 2) (l1)
6	Mast Adjustment	Layer 5. Upper link limits	*ASAE Standards,* S217.12 DEC01 (p. 83, 3.2.22)

adjustment heights, and lower hitch point height. These are the first design decisions made, but they can be revised at any stage of the design process. All elements of Layer 2 are shown in figure 2.

Layer 2 has four horizontal lines representing the actual positions of the four parameters. These lines are constrained to be horizontal and are not Dimension Locked. The designer can click and drag these lines in the vertical direction to set the heights, allowing for quick and easy changes to these parameters at any point in the design process. If the designer prefers to establish these parameters numerically, vertical dimensions between the ground and the lines can be used to Dimension Lock the values, in which case parameter changes are made by altering the values of the dimensions. In either case, it is immediately obvious if the parameters chosen conform to the limits imposed by the standard. For example, the line representing the actual transport height must be either on or above the line representing the minimum transport height, drawn in Layer 1. The designer gets an explicit graphical representation of the boundaries and the permissible zones for the performance parameters. GCP thus works to bring out the freedom within the standard and place it at the disposal of the designer.

Layer 3: Mechanism Side View

In Layer 3, the side view of the three-point hitch as a planar four-bar linkage and associated constraints are included. All elements of Layer 3 are shown in figure 3. The designer's objective is to determine the positions of link points A1 and B1 (defined in fig. 1), lengths of the lower and upper links, transport pitch and vertical convergence distance. The upper link must be of variable length to satisfy the actual mast adjustment. Its maximum and minimum lengths are determined later in Layer 5.

The four-bar linkage is defined by the lower link A1A2, mast or coupler A2B2, upper link B2B1 and fixed link A1B1 on the tractor body. The linkage is drawn in two positions. In the first (solid lines in fig. 3), the lower link is horizontal, and the mast vertical. The second (dashed lines in fig. 3) shows the linkage raised to the actual transport height.

Points A1 and B1 are placed on the diagram by drawing two circles of small arbitrary radius. The centers of these circles represent points A1 and B1. These points are not Dimension Locked, and their position can be changed during the design process. The mechanism in the first position is completed by drawing lines A1A2, A2B2, and B2B1. Point A2 is located arbitrarily. Line A1A2 is constrained to be horizontal, and line A2B2 is constrained to be vertical. A2B2 is Dimension Locked to have a value within the narrow

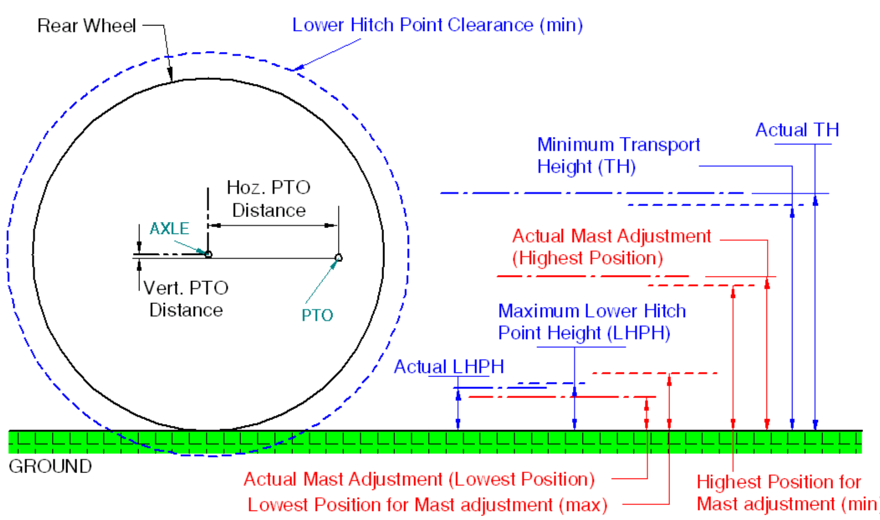

Figure 2. Side view showing the rear tire of a tractor and the constraints imposed on the hitch. All elements of Layers 1 and 2 are shown.

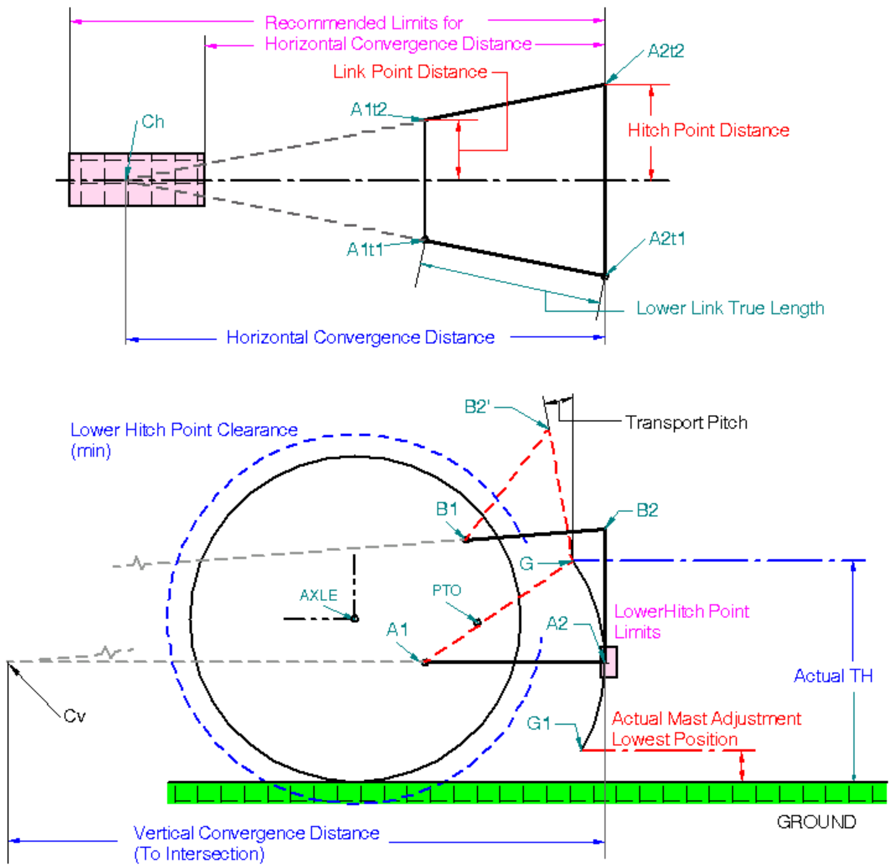

Figure 3. Three-point hitch as a planar four-bar linkage, side and top views. All elements of Layers 1, 3, and 4 and only the relevant elements of Layer 2 are shown.

tolerance zone defined for the mast height. Due to the small acceptable range, the mast height is not considered a design variable. Note that the corresponding endpoints of the three drawn links of the mechanism are Strong Connected because each is drawn from the endpoint of another. This forms pin joints at points A2 and B2.

An arc of radius A1A2 centered at A1 represents the path traveled by point A2 as the hitch is operated. One end is Weak Connected to the actual transport height line (point G), and the other to the line representing the actual lowest position for mast adjustment (point G1). The linkage in the second position is completed by drawing lines A1G, GB2' and B2'B1 and applying Equality constraints on the corresponding sides in the two positions (A1A2 = A1G, A2B2 = GB2', B2B1 = B2'B1).

The standard specifies acceptable zones for the location of lower hitch point A2, transport pitch at G and mast adjustment at G1 (fig. 4). A rectangle represents the permissible zone for the location of point A2 when the lower link is horizontal (fig. 3). The distances between the vertical sides of the rectangle and the end of the PTO shaft are Dimension Locked (table 2, No. 3). The height of this rectangle, being of no particular consequence, is Dimension Locked such that the rectangle remains aligned with point A1 as it moves during manipulation of the mechanism. The designer can thus see if lower hitch point A2 is within the permissible range at any given moment. The transport pitch angle is constructed by drawing a vertical line from point G and placing an angular dimension between B1B2' and the vertical. If this is Dimension Locked, a solution tailored for a particular transport pitch is obtained, but if not, the value is free to change as additional constraints are imposed and other parameters varied. In the latter case, the designer must verify that the value remains within the standard's limits, so it is essential that this dimension be placed on the same layer as the mechanism.

The vertical convergence point is constructed by drawing lines A1Cv and CvB1 and constraining them to be parallel to lines A1A2 and B1B2, respectively. A general dimension from A2 to Cv is placed in this layer to indicate the vertical convergence distance. The standard recommends that this distance be greater than 0.9 times the tractor's wheel base. The designer can verify that this condition is satisfied or Dimension Lock the value to achieve solutions with this added constraint.

The following describes the manipulations that can be accomplished within Layer 3 to exploit the freedom provided by the standard. The mechanism can be manipulated by clicking and dragging any of the points A1, A2, B1, B2, G, and Cv, any of the lines A1A2, A2B2, B2B1, A1Cv, and CvB1, the arc A2G and the lines representing transport height and lower hitch point height. The mechanism is dynamically updated as any of these entities are moved. Provided that point A2 is within the permissible zone, each position of the mechanism represents a design solution satisfying all chosen values for the various parameters. Further, if any of the dimensions are converted from general to locked dimensions (or vice-versa), the manipulation of the mechanism dynamically generates solutions satisfying the added (or reduced) constraints. In particular, when dimensions for performance parameters are fixed, solutions tailored for the selected values are obtained. If the characteristic features of a particular tractor define specific areas within which the link points need to be placed, these areas can be indicated in Layer 3. One of the link points may be Position Locked inside such an area, and the other link point may be moved to check for solutions. In general, any of the points or links can be Position Locked to impose additional constraints. For example, once a solution for a near parallel lift of the mast (±3° transport pitch) is obtained, the lower link can be locked, and the position of B1 can be varied to obtain the position of the second upper link point for getting a transport pitch in the second range (+10° to +15°) specified by the standard. The vertical convergence distance, upper link point (and hence upper link length), and transport height have strong interrelationships. As the transport height increases beyond the minimum required transport height, the convergence distance approaches infinity, and then flips so that convergence takes place behind the tractor. Also, if the vertical convergence distance is decreased, the upper link length increases.

Without imposing additional constraints, the diagram has more than two degrees of freedom. In such underconstrained diagrams, infinitely many positions of related elements (solutions) exist for a given displacement of an element. The software uses optimization strategies within its constraint manager, to which the designer has no access, to move some of the related elements and hold others fixed. The constraint manager thus imposes additional constraints so that the solutions appearing on the screen are a subset of all possible solutions for the given displacement. The designer can control this only by imposing additional Position or Dimension Locks and reducing the degrees of freedom of the diagram. For example, as the vertical convergence distance is changed, the diagram may update by changing the position of point B1. If point B1 is Position Locked, the diagram updates by changing the position of point A2. Understanding this behavior of the kinematic diagram is part of the designer's intuition development.

Layer 4: Mechanism Top View

Layer 4 contains the top view of the mechanism and is shown in figure 3. The designer establishes the horizontal convergence distance, and the position of the lower link points in the horizontal plane is obtained. The true length of the lower links is determined with the lower links horizontal when viewed in the vertical plane. These dimensions, along with those obtained from the dimensioning layer associated with the 'Mechanism Side View' Layer (Layer 3), provide the complete spatial description of the link and hitch points. Along with the upper link length range (obtained in Layer 5), the length of the lower links, and the specified mast height, the three-point hitch is completely defined.

The top view is constructed using the lower hitch and link points. The upper link is redundant in this view and not drawn. Points A1 and A2 are projected in the top view as A1t1, A1t2 and A2t1, A2t2 ('t' stands for 'Top View'). The lower hitch points A2t1 and A2t2 are placed on the projector from A2 symmetrically about a center line and Dimension Locked (table 2, No. 5). Lines A2t1-A1t1 and A2t2-A1t2 representing the lower links are drawn and constrained to be Equal. The horizontal convergence point, Ch, is constructed in a manner similar to the vertical convergence point. A rectangle constrained using Dimension Locks (table 1, No. 7) indicates the standard's recommended range of the horizontal convergence distance. A general dimension for the horizontal convergence distance is placed in Layer 4 which can be Dimension Locked to tailor the value for this performance parameter.

Since the side and top views are linked, changes in one view affect the configuration of the linkage in the other. To avoid unnecessary iterations due to this coupling, it is advisable to fix as many parameters as possible in the side view before constructing the top view. The side view has more variables and is relatively insensitive to the changes in the top view. Ideally, the designer would establish only the horizontal convergence distance and the lower link point position in the top view.

Layer 5: Upper Link Limits

Layer 5 provides the minimum and maximum lengths that the upper link must achieve so that the mechanism can achieve ±5° pitch for the mast adjustment specified in Layer 2 (fig. 4). Layer 5 provides output based on the work done in Layers 2 and 3, but can easily be used to impose additional constraints by Dimension Locking any of the upper link lengths.

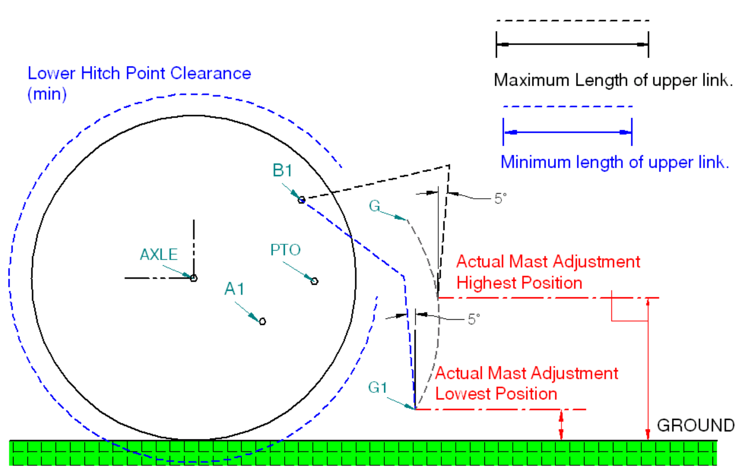

Figure 4. Upper link limits for the current hitch configuration. All elements of Layers 1 and 5 and the relevant elements of Layers 2 and 3 are shown.

The two mast positions are constructed at points G and G1 and Dimension Locked to be oriented 5° clockwise and counter-clockwise from the vertical. The endpoints of the mast in the two positions are connected to point B1 to form the upper link in each configuration. Two horizontal lines, shown in the upper right corner of figure 4, are constrained to be Equal to the upper link in each of the positions. General dimensions of these lines provide the required maximum and minimum upper link lengths.

Layer 6: Movement Range

The movement range and the leveling adjustment are the performance parameters that define the working range of the hitch. Layer 6 allows the designer to specify these parameters and compare them with the required minimum values specified by the standard (fig. 5). The driving four-bar is configured so that these performance parameters of the hitch are satisfied.

Layer 6 contains rectangles that specify the required minimum depths of the parameters below the actual transport height for the movement range and the leveling adjustment (table 1, Nos. 8, 9). The lower link is drawn again in this layer from point A1 to represent three positions: the beginning and end of the movement range and the end of the leveling adjustment. The parameters can be chosen by manipulating the lower links. This manipulation also affects the power travel of the driving crank CD of the driving four-bar. The specified leveling adjustment determines the extension required in the length of arm DE. The standard influences the configuration of the driving linkage through the limiting values of the movement range and the leveling adjustment.

Layer 7: Driving Four-Bar

Layer 7 contains the side view of the driving four-bar linkage CDEA1 (fig. 5), wherein the designer establishes the positions of the points C, D, and E. The lift of crank CD and the actual transport height are obtained from this layer. The position of point A1 is obtained from Layer 3. Since link CD is the actuated link, point C is connected at a power source on the tractor, and hence, its location is governed by the design of the tractor. Point E lies on the lower link A1A2. The positions of points D and E determine the mechanical advantage and affect the lifting capacity of the driving four-bar. The lengths of links CD, DE, and A1E may thus be decided based on other considerations. The lift of crank CD, which is also the rotation of the drive shaft at point C, is a kinematic quantity to be established such that the hitch can traverse the specified movement range.

The driving four-bar is in a plane defined by link A1A2 and line A1B1, which is not parallel to the vertical longitudinal plane (fig. 1). Hence, the links CD and DE in the side view are not the true lengths. Appropriate apparent lengths are used in figure 5. Geometrical constructions can be added to the diagram to obtain true lengths from apparent lengths of the links. Alternatively, the same constructions can be used to get the apparent lengths of the links if the true lengths are already known.

The driving four-bar is drawn in two positions such that link A1E is at the two extremes of the specified movement range. Point E is located on the lower link using a Weak Connect and a Dimension Lock. A general angular dimension between the two positions of crank CD measures the lift. This angle is dynamically updated as any of the points C, D, and E or the actual transport height and movement range values are changed. The leveling adjustment is achieved by changing the length of lift rod DE. The diagram can give the change in lift rod length required to obtain the current leveling adjustment, again an apparent length that may be converted to a true length.

Discussion and Results

This section presents different ways that the diagram can be used for design purposes. An example is provided to illustrate the application of the technique to an existing hitch configuration.

Use of GCP for Hitch Design

Although the presented sequence is useful for creating the kinematic diagram, actual design can be iterative, with the designer modifying previous decisions whenever required. The software assists the designer by instantaneously updating the diagram. For example, the lower hitch point height influences vertical convergence distance, so the designer may return to Layer 2 and manipulate the lower hitch point height to sufficiently increase vertical convergence distance. Further, GCP facilitates iteration between the kinematic and dynamic or other aspects of hitch design. For example, if

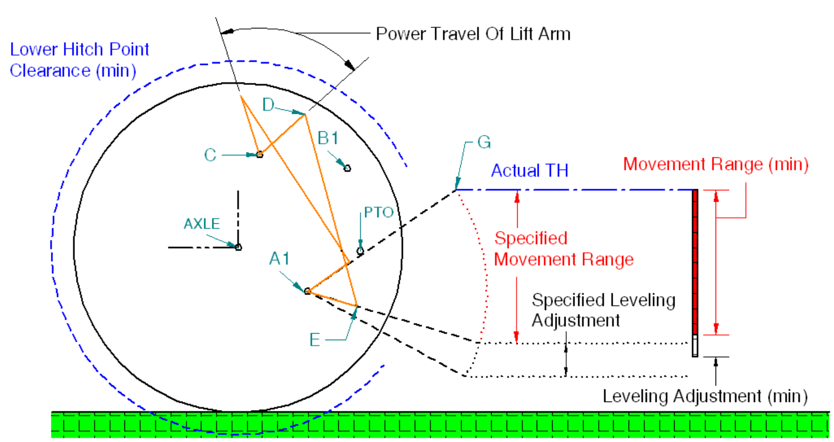

Figure 5. Driving four-bar linkage, movement range, and leveling adjustment. All elements of Layers 1 and 6 and 7 and the relevant elements of Layers 2 and 3 are shown.

some dynamic concern requires a modification, the designer can implement the change in this layout and immediately see if the change causes the mechanism to violate any limits of the standard.

The real-time updating aspect of GCP can also be used to animate kinematic diagrams that have at least one degree of freedom. If one link is dragged utilizing one degree of freedom, the diagram updates to satisfy the imposed constraints, displaying the motion of the mechanism. The kinematic diagram can be used to validate an existing three-point hitch design if defined using measured dimensions. The diagram indicates if the position of point A2 is within the specified limits. Then, the mechanism can be animated and performance parameters like transport height, transport pitch, and lower hitch point height for maximum travel of crank CD can be compared to the requirements of the standard. The mast adjustment possible with the available upper link length variation can be checked.

Example Using GCP for Hitch Design

The authors have chosen the New Holland model TL 70 Diesel tractor with a category two hitch to demonstrate the efficacy of the technique. Configurations representing the mechanism on the actual tractor and a new design were created. The dimensions of the existing mechanism were obtained from online sources. The techniques described in this article were applied to generate the new design that demonstrated improved performance in terms of transport height. The following design decisions were made to obtain the alternative design, and the corresponding values are listed in tables 3 and 4.

In Layer 2, the lower hitch point height and the mast adjustment heights were fixed as 18, 65, and 14 cm, respectively. The horizontal convergence distance was fixed at 215.07 cm in Layer 3. The movement range and leveling adjustment were fixed at 92.12 and 15.4 cm, respectively, in Layer 6. The position of point B1 was changed, although this did not take into account the specific tractor geometry. The purpose of the example is simply to investigate how the freedom within the standard might be exploited. Table 4 gives the output dimensions for the new configuration and the existing design. The upper and lower link lengths, the positions of points A1 and B1 and the maximum transport height are determined. The power travel of the lift is also determined.

The maximum transport height for the existing configuration is 108.33 cm. The maximum transport height for the designed mechanism is 127.52 cm. These values were determined by fixing the transport pitch at the highest permissible value of +15°. Better performance in terms of transport height was achieved with only minor changes to other performance parameters as described below.

The kinematic diagram used to generate the alternative solution is underconstrained, so the presented result represents one of many possible solutions. The significant rise in

Table 3. Dimensions for the TL 70 D (New Holland).

No.	Dimension	ASAE Imposed Limit	Model Dimension	Selected Values for the Designed Configuration
1	Diameter of rear tire	–	148.34 cm	148.34 cm
2	Minimum transport height	95 cm	–	–
3	Maximum lower hitch point height	20 cm	–	18 cm
4	Mast adjustment heights	61 cm (min), 20 cm (max)	–	65 cm and 14 cm
5	Lower hitch point clearance	10 cm	–	–
6	Location dimensions of the PTO shaft from the axle center	–	55 cm horizontal, 1.5 cm below	–
7	Permissible range for the length of the lower link in the side view	55cm to 62.5 cm from the end of the PTO shaft	–	–
8	Mast height	61 cm	–	–
9	Movement range	65 cm	–	92.12 cm
10	Leveling adjustment	10 cm	–	15.4 cm
11	Location dimensions of the lower hitch points in the top view	87.4 cm	–	–
12	Recommended range for the horizontal convergence distance	180 cm to 240 cm	–	215.07 cm

Table 4. Existing and modified design parameters for the TL 70 D (New Holland).

No.	Design Parameter	Designed Configuration	Existing Configuration
1	Maximum transport height for 15° transport pitch	127.52 cm	108.33 cm
2	Hitch point A1	20.17 cm below and 13.5 cm behind the Axle Center and 23.9 cm from the center line (in top view)	20.17 cm below and 13.5 cm behind the Axle Center and 23.9 cm from the center line (in top view)
3	Hitch point B1	40.83 cm above and 33.85 cm behind the Axle Center	33.5 cm above and 40 cm behind the Axle Center
4	Length of the lower link	96.5 cm	96.5 cm
5	Length of the upper link	74.04 cm (min), 81.56 cm (max)	70.38 cm (with a horizontal lower link and a vertical mast)
6	Power travel of the lift	155°	–
7	Vertical convergence distance	9695.11 cm (greater than 0.9 times wheelbase)	582.38 cm

the transport height is obtained by making the upper and the lower links almost parallel. This accounts for the very large vertical convergence distance noted in table 4. The increased transport height makes it necessary to increase the movement range and leveling adjustment so that the lower hitch point can get close enough to the ground to satisfy the lower hitch point height requirement. Parallel lift of the mast can be achieved if the upper and lower links are made parallel and equal in length. The mechanism then is a parallelogram at every point in the movement range and can achieve a 0° transport pitch for any transport height. The lift of the hitch is then limited by the lower hitch point clearance limit, which prevents the lower hitch point from coming too close to the rear wheel of the tractor. Thus, there seems to be a constraint on the position of point B1 with respect to A1 for obtaining the required transport height with a near parallel lift of the mast. It was also observed that for a transport pitch of 10°, the vertical convergence takes place behind the tractor if the transport height is increased much beyond the minimum requirement. This suggests that the layout becomes quite sensitive to variations in the transport height when the transport pitch is fixed. In general, the layout becomes increasingly sensitive to variations in an element as more performance parameters or positions of points are fixed.

Application and Advantages of GCP

Apart from the advantages of GCP already mentioned, there are other benefits specific to its application to three-point hitch design. First, when the kinematic constraints imposed on a mechanism are non-standard, 'implied' constraints may exist, such as the limit on the relative positions of A1 and B1 for a near parallel lift of the mast. Geometric insights such as implied constraints and knowledge of the sensitivity of different design variables have the potential to influence the design of not just the hitch, but also the implements, PTO shaft, and tractor body.

Another benefit of the technique is that the foundational approach presented in this article can be easily expanded to include the geometry of the tractor. Additional constraints based on the shape and size of a particular model can be incorporated in the diagram to get hitch configurations tailored to meet the requirements of both the governing standard and the specific tractor geometry. The quick couplers used for heavy implements, governed by the *ASAE Standards (*2003), can also be incorporated into the diagram. Appropriate additions to the described diagram can be made to obtain a complete description of the driving four-bar linkage. The diagram can also include connecting elements like the implement input driveline (*ASAE Standards*, 2004) and the implement itself. This would allow the designer to study the kinematic aspects of the expanded system, such as variation in length of the implement input driveline shaft as the hitch is raised to the transport height and the relationship between the hitch configuration and the implement depth. This idea can be extended further, to construct a model of the tractor, hitch, and implement that can be constrained (Weak Connected) to move along an uneven ground to study interactions between the articulated elements.

With practice, the designer is able to make accurate guesses about the possible positions of various link and hitch points, and with creativity in imposing additional constraints, the process of manipulating the diagram can be made even more efficient. The technique serves as an effective aid to analytical kinematic synthesis approaches and helps develop intuitive skill. In working with the kinematic layout, the designer develops a feel for the freedom available for further manipulation within the limits of the standard. Such manipulation may be necessitated by dynamic or other concerns, and iteration between the kinematic and dynamic aspects of design is facilitated by this technique. Reduction in cycle time is anticipated due to the increased flexibility in the design process.

Summary

This article proposes the application of geometric constraint programming (GCP) to the kinematic design of three-point hitches used with agricultural tractors. A procedure for constructing a kinematic diagram for designing a hitch is outlined, and a template is used to generate a design for a particular tractor model. The diagrams presented in this article are drawn using SolidEdge (2005), but the technique is generally applicable in most commercially available parametric CAD software packages. GCP gives a comprehensive graphical representation of the large and complex constraint set imposed by the governing *ASAE Standards* (2001). The characteristic of dynamic updating of the kinematic diagram as parameters are varied can provide useful geometric insight and help in the development of intuition. The technique can be used for the kinematic design of three-point hitches in a flexible, efficient manner at no additional software cost.

References

ASAE Standards, 48th ed. 2001. S217.12. Three-point free link attachment for hitching implements to agricultural wheel tractors. St. Joseph, Mich.: ASAE.

ASAE Standards, 50th ed. 2003. S278.7. Agricultural tractors and implements – Three-point hitch couplers – Part 1: U-frame coupler. St. Joseph, Mich.: ASAE.

ASAE Standards, 51st ed. 2004. S207.12. Operating requirements for tractors and power take-off driven implements. St. Joseph, Mich.: ASAE.

Cordesses L., J. P. Poirier, and C. Veron. 2002. Performance analysis of a three-point hitch controller. In *Proc. IEEE/RSJ Intl. Conf. Intelligent Robots Syst*., 2233-2238. Lausanne, Switzerland: IEEE.

Garner, T. H., R. B. Dodd, D. Wolf, and U. M. Peiper. 1988. Force analysis and application of a three-point hitch dynamometer. *Transactions of the ASAE* 31(4): 1047-1053.

Kinzel, E. C., J. P. Schmiedeler, and G. R. Pennock. 2006. Kinematic synthesis for finitely separated positions using geometric constraint programming. *ASME J. Mech. Design* 128(5): 1070-1079

Lang, T., and H. H. Harms. 2002: A new concept for the three-point hitch as a mechatronic system. In *Automation Technology for Off-Road Equipment*, 246-251, Chicago, Ill.: ASAE.

Morling, R. W. 1979. Agricultural tractor hitches, analysis of design requirements. In ASAE Distinguished Lecture Series – Tractor Design No. 5. St. Joseph, Mich.: ASAE.

SolidEdge. 2005. Ver. 17.00.05.03. Planto, Tex.: UGS.

Waldron, K. J., and G. L. Kinzel. 2004. *Kinematics, Dynamics, and Design of Machinery,* 2nd ed. New York: John Wiley and Sons.